Real Numbers and Algebraic Expressions
The set \{1, 3, 5, 7, 9\} has five elements.

- A set is a collection of objects whose contents can be clearly determined.
- The objects in a set are called the elements of the set.
- We use braces to indicate a set and commas to separate the elements of that set.

For example,

The set of counting numbers can be represented by \{1, 2, 3, \ldots \}.
The set of even counting numbers are \{2, 4, 6, \ldots \}.

The set of even counting numbers is a subset of the set of counting numbers, since each element of the subset is also contained in the set.
## Important Subsets of the Real Numbers

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Examples</th>
</tr>
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<tbody>
<tr>
<td><strong>Natural Numbers</strong>&lt;br&gt;<code>N</code></td>
<td><code>{1, 2, 3, …}</code>&lt;br&gt;<em>These are the counting numbers</em></td>
<td>4, 7, 15</td>
</tr>
<tr>
<td><strong>Whole Numbers</strong>&lt;br&gt;<code>W</code></td>
<td><code>{0, 1, 2, 3, …}</code>&lt;br&gt;<em>Add 0 to the natural numbers</em></td>
<td>0, 4, 7, 15</td>
</tr>
<tr>
<td><strong>Integers</strong>&lt;br&gt;<code>Z</code></td>
<td><code>{…, -2, -1, 0, 1, 2, 3, …}</code>&lt;br&gt;<em>Add the negative natural numbers to the whole numbers</em></td>
<td>-15, -7, -4, 0, 4, 7</td>
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### Important Subsets of the Real Numbers

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<td><strong>Rational Numbers</strong></td>
<td><em>These numbers can be expressed as an integer divided by a nonzero integer: Rational numbers can be expressed as terminating or repeating decimals.</em></td>
<td>$-17 = \frac{-17}{1}, -5 = \frac{-5}{1}, -3, -2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0, 2, 3, 5, 17$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{2}{5} = 0.4,$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{-2}{3} = -0.666666... = -0.\overline{6}$</td>
</tr>
<tr>
<td><strong>Irrational Numbers</strong></td>
<td><em>This is the set of numbers whose decimal representations are neither terminating nor repeating. Irrational numbers cannot be expressed as a quotient of integers.</em></td>
<td>$\sqrt{2} \approx 1.414214$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-\sqrt{3} \approx -1.73205$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\pi \approx 3.142$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-\frac{\pi}{2} \approx -1.571$</td>
</tr>
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</table>
The set of real numbers is formed by combining the rational numbers and the irrational numbers.
The real number line is a graph used to represent the set of real numbers. An arbitrary point, called the origin, is labeled 0;

Units to the left of the origin are negative.

Units to the right of the origin are positive.
Real numbers are graphed on the number line by placing a dot at the location for each number. –3, 0, and 4 are graphed below.
On the real number line, the real numbers increase from left to right. The lesser of two real numbers is the one farther to the left on a number line. The greater of two real numbers is the one farther to the right on a number line.

Since 2 is to the left of 5 on the number line, 2 is less than 5. \(2 < 5\)
Since 5 is to the right of 2 on the number line, 5 is greater than 2. \(5 > 2\)
Because $-5 = -5 - 5 > -5$.
Because $7 > 37 > 3$.

**Inequality Symbols**

<table>
<thead>
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<th>Symbols</th>
<th>Meaning</th>
<th>Example</th>
<th>Explanation</th>
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<tr>
<td>$a \leq b$</td>
<td>$a$ is less than or equal to $b$.</td>
<td>$3 \leq 7$</td>
<td>Because $3 &lt; 7$</td>
</tr>
<tr>
<td>$b \geq a$</td>
<td>$b$ is greater than or equal to $a$.</td>
<td>$7 \geq 3$</td>
<td>Because $7 &gt; 3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-5 \geq -5$</td>
<td>Because $-5 = -5$</td>
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Absolute Value

Absolute value describes the distance from 0 on a real number line. If $a$ represents a real number, the symbol $|a|$ represents its absolute value, read “the absolute value of $a$.”

For example, the real number line below shows that $|-3| = 3$ and $|5| = 5$.

The absolute value of –3 is 3 because –3 is 3 units from 0 on the number line. The absolute value of 5 is 5 because 5 is 5 units from 0 on the number line.
Definition of Absolute Value

The absolute value of $x$ is given as follows:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$
Properties of Absolute Value

For all real numbers $a$ and $b$,

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $a \leq |a|$

4. $|ab| = |a||b|$

5. $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$, $b$ not equal to 0

6. $|a + b| \leq |a| + |b|$ (the triangle inequality)
Example

• Find the following: |-3| and |3|.

Solution:

\[-3| = 3 \text{ and } |3| = 3\]
Distance Between Two Points on the Real Number Line

If $a$ and $b$ are any two points on a real number line, then the distance between $a$ and $b$ is given by

$|a - b|$ or $|b - a|$
Find the distance between –5 and 3 on the real number line.

**Solution**  Because the distance between a and b is given by \(|a – b|\), the distance between –5 and 3 is \(|-5 – 3| = |-8| = 8\).

We obtain the same distance if we reverse the order of subtraction:

\[|3 – (-5)| = |8| = 8.\]
Algebraic Expressions

A combination of variables and numbers using the operations of addition, subtraction, multiplication, or division, as well as powers or roots, is called an algebraic expression.

Here are some examples of algebraic expressions:

\[ x + 6, \quad x - 6, \quad 6x, \quad x/6, \quad 3x + 5. \]
The Order of Operations Agreement

1. Perform operations within the innermost parentheses and work outward. If the algebraic expression involves division, treat the numerator and the denominator as if they were each enclosed in parentheses.
2. Evaluate all exponential expressions.
3. Perform multiplication or division as they occur, working from left to right.
4. Perform addition or subtraction as they occur, working from left to right.
Text Example

The algebraic expression $2.35x + 179.5$ describes the population of the United States, in millions, $x$ years after 1980. Evaluate the expression when $x = 20$. Describe what the answer means in practical terms.

**Solution**  We begin by substituting 20 for $x$. Because $x = 20$, we will be finding the U.S. population 20 years after 1980, in the year 2000.

$$
2.35x + 179.5 \\
= 2.35(20) + 179.5 \\
= 47 + 179.5 \\
= 226.5
$$

Thus, in 2000 the population of the United States was 226.5 million.
# Properties of the Real Numbers

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<tr>
<td>Commutative Property of Addition</td>
<td>Two real numbers can be added in any order.</td>
<td>• $13 + 7 = 7 + 13$</td>
</tr>
<tr>
<td></td>
<td>$a + b = b + a$</td>
<td>• $13x + 7 = 7 + 13x$</td>
</tr>
<tr>
<td>Commutative Property of Multiplication</td>
<td>Two real numbers can be multiplied in any order.</td>
<td>• $x \cdot 6 = 6x$</td>
</tr>
<tr>
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<td>$ab = ba$</td>
<td></td>
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<tr>
<td>Associative Property of Addition</td>
<td>If 3 real numbers are added, it makes no difference which 2 are added first.</td>
<td>• $3 + (8 + x)$</td>
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<td>$(a + b) + c = a + (b + c)$</td>
<td>= $(3 + 8) + x$</td>
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<tr>
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<td>= $11 + x$</td>
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| Associative Property of Multiplication | If 3 real numbers are multiplied, it makes no difference which 2 are multiplied first.  
  \((a \cdot b) \cdot c = a \cdot (b \cdot c)\)                  | \(-2(3x) = (-2 \cdot 3)x = -6x\)                                      |
| Distributive Property of Multiplication over Addition | Multiplication distributes over addition.  
  \(a \cdot (b + c) = a \cdot b + a \cdot c\)                   | \(5 \cdot (3x + 7)\)  
  \(= 5 \cdot 3x + 5 \cdot 7\)  
  \(= 15x + 35\)                                      |
| Identity Property of Addition       | Zero can be deleted from a sum.  
  \(a + 0 = a\)  
  \(0 + a = a\)                                               | \(0 + 6x = 6x\)                                                      |
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<td>Identity Property of Multiplication</td>
<td>One can be deleted from a product. $a \cdot 1 = a$ and $1 \cdot a = a$</td>
<td>$\cdot 1 \cdot 2x = 2x$</td>
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<tr>
<td>Inverse Property of Addition</td>
<td>The sum of a real number and its additive inverse gives 0, the additive identity. $a + (-a) = 0$ and $(-a) + a = 0$</td>
<td>$\cdot (-6x) + 6x = 0$</td>
</tr>
<tr>
<td>Inverse Property of Multiplication</td>
<td>The product of a nonzero real number and its multiplicative inverse gives 1, the multiplicative identity. $a \cdot 1/a = 1$ and $1/a \cdot a = 1$</td>
<td>$\cdot 2 \cdot 1/2 = 1$</td>
</tr>
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Definitions of Subtraction and Division

Let a and b represent real numbers.

**Subtraction:** $a - b = a + (-b)$
We call $-b$ the **additive inverse** or **opposite** of $b$.

**Division:** $a \div b = a \cdot \frac{1}{b}$, where $b \neq 0$
We call $\frac{1}{b}$ the **multiplicative inverse** or **reciprocal** of $b$. The quotient of $a$ and $b$, $a \div b$, can be written in the form $a/b$, where $a$ is the **numerator** and $b$ the **denominator** of the fraction.
Text Example

Simplify: $6(2x - 4y) + 10(4x + 3y)$.

Solution

$$6(2x - 4y) + 10(4x + 3y)$$

$$= 6 \cdot 2x - 6 \cdot 4y + 10 \cdot 4x + 10 \cdot 3y$$

Use the distributive property.

$$= 12x - 24y + 40x + 30y$$

Multiply.

$$= (12x + 40x) + (30y - 24y)$$

Group like terms.

$$= 52x + 6y$$

Combine like terms.
Properties of Negatives

Let a and b represent real numbers, variables, or algebraic expressions.

1. \((-1)a = -a\)
2. \((-(-a)) = a\)
3. \((-a)(b) = -ab\)
4. \(a(-b) = -ab\)
5. \(-(a + b) = -a - b\)
6. \(-(a - b) = -a + b = b - a\)
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